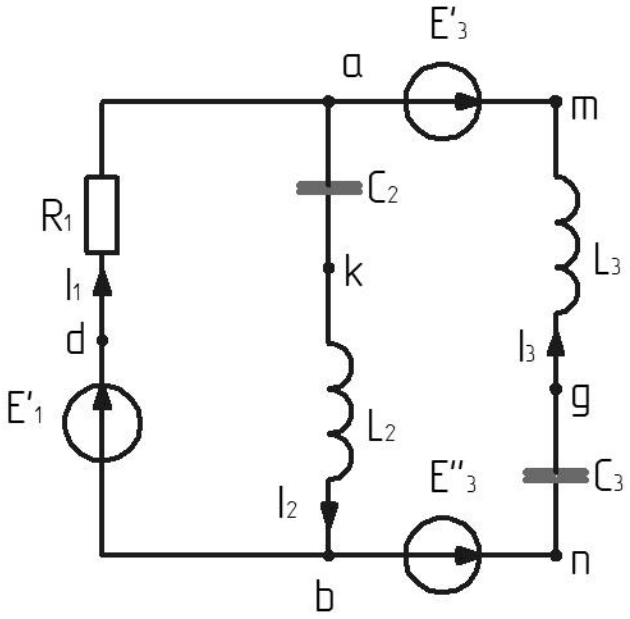


$L_2 = 200 \cdot 10^{-3}$ Гн

$L_3 = 90 \cdot 10^{-3}$ Гн

$C_2 = 200 \cdot 10^{-6}$ Ф

$C_3 = 100 \cdot 10^{-6}$ Ф

$R_1 = 40$ Ом

$f = 50$ Гц

$e'_1 = 100 \cdot \sqrt{2} \cdot \cos(\omega t)$ В

$e'_3 = 120 \cdot \sqrt{2} \cdot \sin(\omega t + 135\text{deg})$ В

$e''_3 = 200 \cdot \sqrt{2} \cdot \cos(\omega t + 315\text{deg})$ В

Параметры схемы в комплексной форме

$\omega = 2\pi f = 314.159$ c^{-1}

$X_{C2} = \frac{1}{j \cdot \omega \cdot C_2} = \frac{1}{j \cdot 314.159 \cdot 200 \cdot 10^{-6}} = -15.915j$ Ом

$X_{C3} = \frac{1}{j \cdot \omega \cdot C_3} = \frac{1}{j \cdot 314.159 \cdot 100 \cdot 10^{-6}} = -31.831j$ Ом

$X_{L3} = j \cdot \omega \cdot L_3 = j \cdot 314.159 \cdot 90 \cdot 10^{-3} = 28.274j$ Ом

$X_{L2} = j \cdot \omega \cdot L_2 = j \cdot 314.159 \cdot 200 \cdot 10^{-3} = 62.832j$ Ом

$E'_1 = 100 \cdot e^{j \cdot (0+90)\text{deg}} = 100j$ В

$E'_3 = 120 \cdot e^{j \cdot 135\text{deg}} = -84.853 + 84.853j$ В

$E''_3 = 200 \cdot e^{j \cdot (315+90)\text{deg}} = 141.421 + 141.421j$ В

1. Составим систему уравнений по законам Кирхгоффа в дифференциальной форме

$$i_1 - i_2 + i_3 = 0$$

$$i_1 \cdot R_1 + L_2 \cdot \frac{d}{dt} i_2 + \frac{1}{C_2} \cdot \int i_2 \, dt = e'_1$$

$$L_3 \cdot \frac{d}{dt} i_3 + \frac{1}{C_3} \cdot \int i_3 \, dt + L_2 \cdot \frac{d}{dt} i_2 + \frac{1}{C_2} \cdot \int i_2 \, dt = e''_3 - e'_3$$

Составим систему уравнений в символической форме

$$I_1 - I_2 + I_3 = 0$$

$$I_1 \cdot R_1 + I_2 \cdot \left(X_{L2} + X_{C2}\right) = E'_1$$

$$I_2 \cdot \left(X_{L2} + X_{C2}\right) + I_3 \cdot \left(X_{L3} + X_{C3}\right) = E''_3 - E'_3$$

Решим систему

$$I_2 = I_1 + I_3$$

$$I_1 \cdot R_1 + \left(I_1 + I_3\right) \cdot \left(X_{L2} + X_{C2}\right) = E'_1$$

$$\left(I_1 + I_3\right) \cdot \left(X_{L2} + X_{C2}\right) + I_3 \cdot \left(X_{L3} + X_{C3}\right) = E''_3 - E'_3$$

$$I_1 \cdot \left(R_1 + X_{L2} + X_{C2}\right) + I_3 \cdot \left(X_{L2} + X_{C2}\right) = E'_1$$

$$I_1 \cdot \left(X_{L2} + X_{C2}\right) + I_3 \cdot \left(X_{L2} + X_{C2} + X_{L3} + X_{C3}\right) = E''_3 - E'_3$$

$$Z_{11} = R_1 + X_{L2} + X_{C2} \text{ explicit, ALL } = 40 + 62.832j + -15.915j \quad \text{Ом}$$

$$Z_{12} = X_{L2} + X_{C2} \text{ explicit, ALL } = 62.832j + -15.915j \quad \text{Ом}$$

$$Z_{21} = X_{L2} + X_{C2} \text{ explicit, ALL } = 62.832j + -15.915j \quad \text{Ом}$$

$$Z_{22} = X_{L2} + X_{C2} + X_{L3} + X_{C3} \text{ explicit, ALL } = 62.832j + -15.915j + 28. \text{Ом} j + -31.831j$$

$$I_1 \cdot Z_{11} + I_3 \cdot Z_{12} = E'_1$$

$$I_1 \cdot Z_{21} + I_3 \cdot Z_{22} = E''_3 - E'_3$$

$$I_1 = \frac{E'_1}{Z_{11}} - I_3 \cdot \frac{Z_{12}}{Z_{11}}$$

$$\left(\frac{E'_1}{Z_{11}} - I_3 \cdot \frac{Z_{12}}{Z_{11}}\right) \cdot Z_{21} + I_3 \cdot Z_{22} = E''_3 - E'_3$$

$$I_3 \cdot \left(Z_{22} - \frac{Z_{12}}{Z_{11}} \cdot Z_{21} \right) = E''_3 - E'_3 - \frac{E'_1}{Z_{11}} \cdot Z_{21}$$

$$I_3 = \frac{E''_3 - E'_3 - \frac{E'_1}{Z_{11}} \cdot Z_{21}}{Z_{22} - \frac{Z_{12}}{Z_{11}} \cdot Z_{21}}$$

$$I_3 = \frac{141.421 + 141.421j - (-84.853 + 84.853j) - \frac{100j}{40 + 62.832 \cdot j + -15.915 \cdot j} \cdot (62.832 \cdot j + -15.915 \cdot j)}{62.832 \cdot j + -15.915 \cdot j + 28.274 \cdot j + -31.831 \cdot j - \frac{62.832 \cdot j + -15.915 \cdot j}{40 + 62.832 \cdot j + -15.915 \cdot j} \cdot (62.832 \cdot j + -15.915 \cdot j)}$$

$I_3 = 7.967 - 5.627j$ A $\quad |I_3| = 9.754$ A $\quad \angle(I_3) = -35.233$

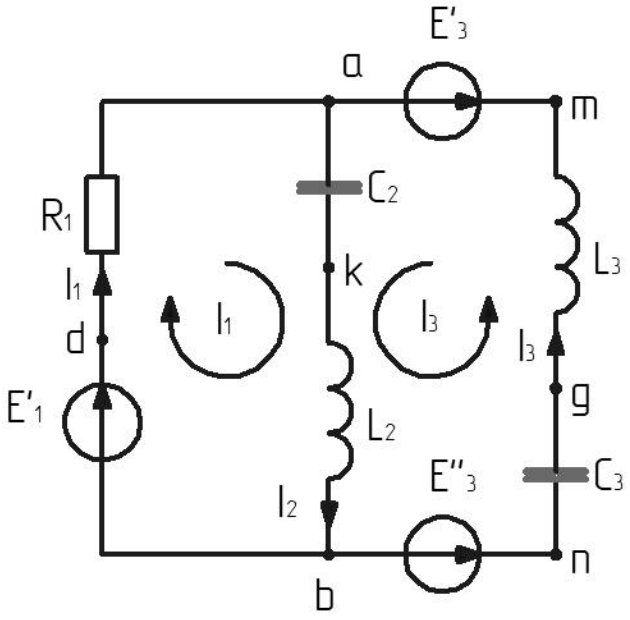
$$I_1 = \frac{E'_1}{Z_{11}} - I_3 \cdot \frac{Z_{12}}{Z_{11}} = \frac{100j}{40 + 62.832 \cdot j + -15.915 \cdot j} - (7.967 - 5.627j) \cdot \frac{62.832 \cdot j + -15.915 \cdot j}{40 + 62.832 \cdot j + -15.915 \cdot j} = -6.15$$

$I_1 = -6.157 + 0.377j$ A $\quad |I_1| = 6.169$ A $\quad \angle(I_1) = 176.492$

$I_2 = I_1 + I_3 = -6.157 + 0.377j + 7.967 - 5.627j = 1.81 - 5.249j$ A

$|I_2| = 5.553$ A $\quad \angle(I_2) = -70.979$

2. Определим токи ветвей методом контурных токов



Составим систему уравнений

$$I_1\cdot\left(R_1 + X_{L2} + X_{C2}\right) + I_3\cdot\left(X_{L2} + X_{C2}\right) = E'_1$$

$$I_1\cdot\left(X_{L2} + X_{C2}\right) + I_3\cdot\left(X_{L2} + X_{C2} + X_{L3} + X_{C3}\right) = E''_3 - E'_3$$

Решим систему

$Z_{11} = R_1 + X_{L2} + X_{C2}\ \text{explicit, ALL} = 40 + 62.832j + -15.915j$ Ом

$Z_{12} = X_{L2} + X_{C2}\ \text{explicit, ALL} = 62.832j + -15.915j$ Ом

$Z_{21} = X_{L2} + X_{C2}\ \text{explicit, ALL} = 62.832j + -15.915j$ Ом

$Z_{22} = X_{L2} + X_{C2} + X_{L3} + X_{C3}\ \text{explicit, ALL} = 62.832j + -15.915j + 28.\ \text{Ом}j + -31.831j$

$$I_1\cdot Z_{11} + I_3\cdot Z_{12} = E'_1$$

$$I_1\cdot Z_{21} + I_3\cdot Z_{22} = E''_3 - E'_3$$

$$I_1 = \frac{E'_1}{Z_{11}} - I_3\cdot\frac{Z_{12}}{Z_{11}}$$

$$\left(\frac{E'_1}{Z_{11}} - I_3\cdot\frac{Z_{12}}{Z_{11}}\right)\cdot Z_{21} + I_3\cdot Z_{22} = E''_3 - E'_3$$

$$I_3\cdot\left(Z_{22} - \frac{Z_{12}}{Z_{11}}\cdot Z_{21}\right) = E''_3 - E'_3 - \frac{E'_1}{Z_{11}}\cdot Z_{21}$$

$$I_3 = \frac{E''_3 - E'_3 - \frac{E'_1}{Z_{11}}\cdot Z_{21}}{Z_{22} - \frac{Z_{12}}{Z_{11}}\cdot Z_{21}}$$

$$I_3 = \frac{141.421 + 141.421j - (-84.853 + 84.853j) - \frac{100j}{40 + 62.832\cdot j + -15.915\cdot j}\cdot(62.832\cdot j + -15.915\cdot j)}{62.832\cdot j + -15.915\cdot j + 28.274\cdot j + -31.831\cdot j - \frac{62.832\cdot j + -15.915\cdot j}{40 + 62.832\cdot j + -15.915\cdot j}\cdot(62.832\cdot j + -15.915\cdot j)}$$

$I_3 = 7.967 - 5.627j$ A $\left|I_3\right| = 9.754$ A $\angle\left(I_3\right) = -35.233$

$$I_1 = \frac{E'_1}{Z_{11}} - I_3\cdot\frac{Z_{12}}{Z_{11}} = \frac{100j}{40 + 62.832\cdot j + -15.915\cdot j} - (7.967 - 5.627j)\cdot\frac{62.832\cdot j + -15.915\cdot j}{40 + 62.832\cdot j + -15.915\cdot j} = -6.15$$

$I_1 = -6.157 + 0.377j$ A $\left|I_1\right| = 6.169$ A $\angle\left(I_1\right) = 176.492$

$I_2 = I_1 + I_3 = -6.157 + 0.377j + 7.967 - 5.627j = 1.81 - 5.249j$ A

$|I_2| = 5.553$ A $\angle(I_2) = -70.979$

3. Определим токи ветвей методом двух узлов. Так как в схеме только два узла то метод сводится к методу двуз узлов

Межузловое напряжение

$$U_{ab} = \frac{\frac{E''_3 - E'_3}{X_{L3} + X_{C3}} + \frac{E'_1}{R_1}}{\frac{1}{X_{L3} + X_{C3}} + \frac{1}{X_{L2} + X_{C2}} + \frac{1}{R_1}} = \frac{\frac{141.421 + 141.421j - (-84.853 + 84.853j)}{28.274j + -31.831j} + \frac{100j}{40}}{\frac{1}{28.274j + -31.831j} + \frac{1}{62.832j + -15.915j} + \frac{1}{40}} = 246.28$$

$U_{ab} = 246.287 + 84.904j$ B $|U_{ab}| = 260.511$ B $\angle(U_{ab}) = 19.021$

Токи ветвей

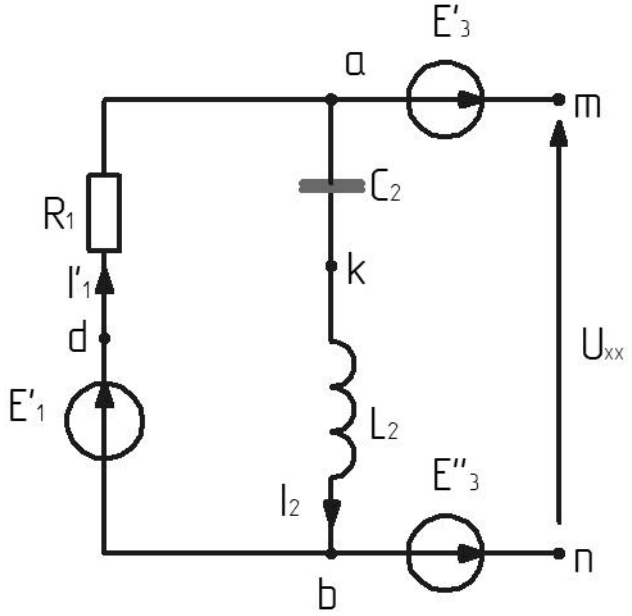
$$I_1 = \frac{E'_1 - U_{ab}}{R_1} = \frac{100j - (246.287 + 84.904j)}{40} = -6.157 + 0.377j$$ A

$$I_2 = \frac{U_{ab}}{X_{L2} + X_{C2}} = \frac{246.287 + 84.904j}{62.832j + -15.915j} = 1.81 - 5.249j$$ A

$$I_3 = \frac{E''_3 - E'_3 - U_{ab}}{X_{L3} + X_{C3}} = \frac{141.421 + 141.421j - (-84.853 + 84.853j) - (246.287 + 84.904j)}{28.274j + -31.831j} = 7.967 - 5.6$$

Результаты совпали

4. Рассчитаем ток ветви 3 методом эквивалентного генератора



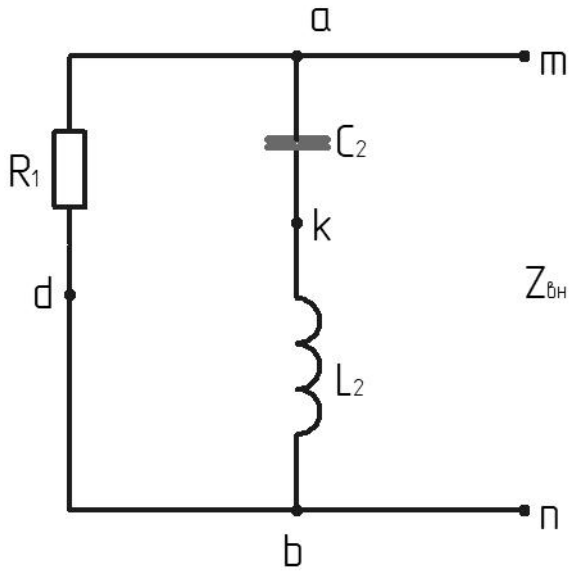
Напряжение холостого хода эквивалентного генератора

$$I'_1 = \frac{E'_1}{R_1 + X_{C2} + X_{L2}} = \frac{100j}{40 - 15.915j + 62.832j} = 1.234 + 1.052j \quad A$$

$$U_{xx} = E''_3 - E'_3 - I'_1 \cdot (X_{C2} + X_{L2})$$

$$U_{xx} = 141.421 + 141.421j - (-84.853 + 84.853j) - (1.234 + 1.052j) \cdot (-15.915j + 62.832j) = 275.645 - 1$$

Внутреннее сопротивление эквивалентного генератора



$$Z_{вн} = \frac{R_1 \cdot (X_{C2} + X_{L2})}{R_1 + X_{C2} + X_{L2}} = \frac{40 \cdot (-15.915j + 62.832j)}{40 - 15.915j + 62.832j} = 23.163 + 19.748j \quad Ом$$

Искомый ток

$$I_3 = \frac{U_{xx}}{Z_{вн} + X_{C3} + X_{L3}} = \frac{275.645 - 1.339j}{23.163 + 19.748j + -31.831j + 28.274j} = 7.967 - 5.627j \quad A$$

Результаты совпали

5. Составим уравнение баланса мощностей и проверим его сходимость

Суммарная мощность источников

$$S_{E'1} = I^*_1 \cdot E'_1 = (100j) \cdot (-6.157 - 0.377j) = 37.74 - 615.718j \quad ВА$$

$$S_{E'3} = I^*_3 \cdot -E'_3 = (7.967 + 5.627j) \cdot -(-84.853 + 84.853j) = 1.153 \times 10^3 - 198.553j \quad ВА$$

$$S_{E''3} = I^*_3 \cdot E''_3 = (141.421 + 141.421j) \cdot (7.967 + 5.627j) = 330.922 + 1.922j \times 10^3 \quad ВА$$

$S_E = S_{E'1} + S_{E'3} + S_{E''3} = 37.74 - 615.718j + 1.153 \times 10^3 - 198.553j + 330.922 + 1.922j \times 10^3$

$S_E = 1.522 \times 10^3 + 1.108j \times 10^3$ BA

Суммарная мощность приемников

$S_1 = I_{1.}{}^2 \cdot R_1 = 6.169^2 \cdot 40 = 1.522 \times 10^3$ BA

$S_2 = I_{2.}{}^2 \cdot (X_{L2} + X_{C2}) = 5.553^2 \cdot (62.832j + -15.915j) = 1.447j \times 10^3$ BA

$S_3 = I_{3.}{}^2 \cdot (X_{L3} + X_{C3}) = 9.754^2 \cdot (28.274j + -31.831j) = -338.355j$ BA

$S = S_1 + S_2 + S_3 = 1.522 \times 10^3 + 1.447j \times 10^3 + -338.355j = 1.522 \times 10^3 + 1.108j \times 10^3$ BA

Как видим $S_E = S = 1.522 \times 10^3 + 1.108j \times 10^3$

6. Сведем результаты в таблицу

	I_1	I_2	I_3
МКТ	-6.157+0.377j	1.81-5.249j	7.967-5.627j
МУП	-6.157+0.377j	1.81-5.249j	7.967-5.627j
МЭГ			7.967-5.627j

7. Рассчитаем напряжения на элементах цепи для построения векторной диаграммы цепи

$U_{L3} = I_3 \cdot X_{L3} = (7.967 - 5.627j) \cdot 28.274j = 159.097 + 225.258j$ B

$|U_{L3}| = 275.777$ B $\angle(U_{L3}) = 54.767$

$U_{C3} = I_3 \cdot X_{C3} = (7.967 - 5.627j) \cdot -31.831j = -179.11 - 253.593j$ B

$|U_{C3}| = 310.467$ B $\angle(U_{C3}) = -125.233$

$U_{L2} = I_2 \cdot X_{L2} = (1.81 - 5.249j) \cdot 62.832j = 329.835 + 113.706j$ B

$|U_{L2}| = 348.885$ B $\angle(U_{L2}) = 19.021$

$U_{C2} = I_2 \cdot X_{C2} = (1.81 - 5.249j) \cdot -15.915j = -83.548 - 28.802j$ B

$|U_{C2}| = 88.373$ B $\angle(U_{C2}) = -160.979$

$U_{R1} = I_1 \cdot R_1 = (-6.157 + 0.377j) \cdot 40 = -246.287 + 15.096j$ B

$|U_{R1}| = 246.749$ B $\angle(U_{R1}) = 176.492$

Построим топографическую диаграмму цепи